Title: Concept Development Unit: Do You Commute? Do You Associate?
The Commutative and Associative Properties and How They Apply to Addition and Multiplication

Brief Overview:

In this unit, students will be introduced to mathematical properties that form the foundation of computation. They will explore the commutative and associative properties of addition and multiplication to discover how order and grouping affect sums and products. Students will build upon their prior knowledge of counting strategies, equality, and the meanings of addition and multiplication. They will use a number line to visualize number relationships and chart the accompanying addition expressions in order to identify patterns and form a generalization about the relationships in the expressions. Using a number generator game, students will work together, creating number sentences to explore the idea that numbers can be added in any order, and the sum will remain the same. Students will act out the story, One Hundred Hungry Ants by Elinor J. Pinczes, generate equations based on the story, explore various grouping formations, and evaluate the affect on sums/products when three or more addends or factors are grouped before they are added or multiplied.

NCTM Content Standard:

Number and Operations:

- Understand numbers, ways of representing numbers, relationships among numbers, and number systems
- Understand meanings of operations and how they relate to one another
- Compute fluently and make reasonable estimates

Algebra:

- Represent and analyze mathematical situations and structures using algebraic symbols
- Use mathematical models to represent and understand quantitative relationships

Grade/Level:

Grades 3 - 5

Duration/Length:

50 minutes

Student Outcomes:

Students will:

- Analyze number relationships and compare equivalent expressions
- Express mathematical relationships using expressions and equations

- Describe and make generalizations about numeric patterns
- Discover flexible ways to add or multiply numbers
- Understand and use properties of operations, such as the commutative and associative properties of addition or multiplication
- Use the commutative and associative properties to compute with whole numbers
- Represent and analyze mathematical situations using symbols, words, and pictures

Materials and Resources:

Lesson 1:

- Teacher Observation Checklist – Counting Strategies (TR1)
- Number line
- Roll An Order (SR1)
- Number Generator
- My Math Journal (SR2)
- Calculator
- Think Sheet for Addition (SR3)
- Commutative Property Think Sheet (SR4)

Lesson 2:

- Teacher Observation Check List – Addition Strategies (TR2)
- Multiplication Picture Stories (TR3)
- Commutative Tile Company Packet Title Page (SR5)
- Floor Plans (SR6)
- Manipulatives such as color tiles
- Customer Recommendation Sheet (SR7)
- Commutative Tile Company Rubric (TR4)
- Two sets of Index Cards (3 x 5 and 4 x 6)
- Multiplication Array Paper (SR8)

Lesson 3:

- Teacher Observation Sheet (TR5)
- <u>One Hundred Hungry Ants</u> (Elinor J. Pinczes)
- Blank Index cards (4x6)
- Manipulatives such as snap cubes, color tiles, or Cuisenaire Rods
- Summative Assessment (SR9)
- Summative Assessment Rubric (TR6)

Development/Procedures:

Lesson 1 - **The Addend Commute**
The Commutative Property, and How It Applies To Addition

Pre-assessment – Pre-assessment will be embedded in the launch activity. Look for the following: *What counting strategies do the students use? Are they comfortable with counting on?* Observe the students as they participate in the launch activity. Use the Teacher Check List (TR1) to record observations.

Launch – Explain that the students will use the "counting on" strategy to get to the next multiple of ten. Choose a number between 2 and 5. Pick a student to mark the number on a number line (number line strips are available commercially or can be drawn on a board or overhead). Pick a second student to "count on" from the chosen number, stopping at ten and marking the ten. Ask the class how they might figure out how many numbers were counted. Using a different color marker, indicate this amount on the number line.

For example, the original number chosen is 3, so choose a student to mark the 3. Then, choose another student to count to ten and mark it. Seven numbers have been counted, so mark each number counted from 3-10 above the number line in an alternate color. In order to reveal the commutative property have the students repeat the process, but this time mark 7 on the number line. Then, mark each number between 7-10, moving beneath the number line in an alternate color. Elicit that a very similar number expression results. Repeat this procedure with numbers ranging from 1 – 100.

Teacher Facilitation – Ask: *Did it matter which way you " counted on" using the number line? Did you arrive at the same number when you finished counting? What did you discover?* Students should verbalize that the order of "counting on" did not matter. Explain that in this lesson they will be introduced to a mathematical property that will help them add numbers and recall basic facts. It is called the Commutative Property. They do not need to memorize the name of the property, but rather, understand its meaning. Next, you might ask: *How do you get to school (car, bus, walk)?* Emphasize that this is called commuting. Explain that they will see how numbers can commute.

Make a class chart of addition fact number sentences.

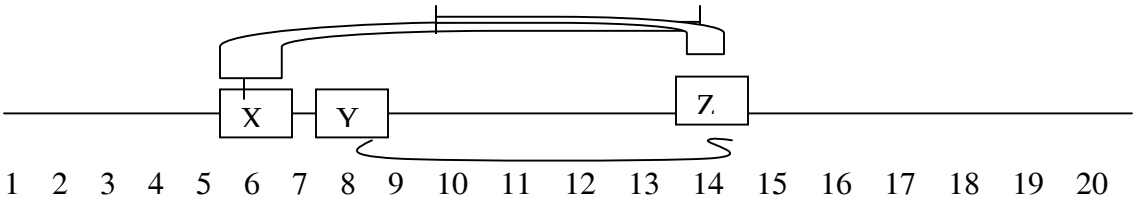
Sample Chart for Grades 2-3:

3 + 2 = 5
8 + 1 = 9
4 + 6 = 10

Sample Chart for Grades 3-5:

6 + 8 = 14
23 + 57 = 80
17 + 3 = 20

Tell the class that they will prove that each number sentence is true by visualizing them on a number line.



Give these instructions: *Using the number line, start at the first number of the equation* (for instance, to represent 6 + 8 =14, the student begins at 6) *and count on the number of times represented by the second number* (ex. 7,8,9,10,11,12,13,14) *to arrive at the sum of the two numbers.* Have another student repeat the procedure, but this time, begin at the second number, count on the equivalent of the first number, thus ending at the sum of the two numbers. Compare the two methods of counting. Ask the class: *How can these two methods of "counting on" be represented as equations?*

Model:

6 + 8 = 14	8 + 6 = 14

How are the equations the same? How are they different? Which method of counting on is easier? Why is it easier?

After charting several fact sentences, ask students to identify any patterns and make a generalization (come up with a rule about the pattern) they think might be true for all addition number sentences.

Student Application – Students will play a game with a partner to enhance their understanding of the usefulness of the commutative property. Pair students and hand out "Roll An Order" (SR1) and a number generator. The sheet includes the following instructions: The first player uses the number generator to find a number for the first box under Player # 1. Player 2 repeats the process, putting their number in the first box under Player # 2. Each player completes her/his number sentence by placing their partner's number in the empty box next to the operation sign. Both players solve their number sentences and compare the sums.

Sample round:

Player #1 rolls 5	**Player #2 rolls 6**
Player #1	Player #2
5 +	6 +
5 + 6	6 + 5
5 + 6 = 11	6 + 5 = 11

Allow 10 – 15 minutes for students to play several rounds. Continue with a "Think, Pair, Share" time. Say: *What patterns do you see in the number sentence pairs? Did our rule*

hold true for all the number combinations in your game? Do you think that the commutative property – adding numbers in any order – will hold true for all addition number sentences? Why or why not? If needed, apply the commutative property to several more examples generated by students.

Embedded Assessment – Students will complete a journal entry. Distribute My Math Journal (SR2). Write the following prompt on the board for students to copy: *What did you discover about ways to make adding two numbers easier? Think about today's game. Use examples from your game to explain how to use mental math strategies to recall basic facts and make addition easier.*

If necessary, give the class a specific example like 6 + 4 and 4 + 6. *Which would be easier to add? Why?* Allow students to assist you in formulating a rubric. Elicit criteria from students that would exemplify a "good" journal response. Possible criteria might include: at least one discovery related to the game; the reasoning behind that idea; an example to support the idea; or a statement that order does not matter when doing addition and a rationale why order does not matter.

Re-teaching – Revisit addition fact families for sums from 1 to 18. Hand out a calculator and the Think Sheet for Addition (SR3). In the "Write the Number Sentences" section, students should write five basic fact sentences in the first column. In the second column, they should reverse the digits, thus, writing a related fact. Next, students will complete the Addition Table by color-coding the chart. They will use a different color to represent each pair of problems. For example, use one color for the coordinates that represent the sums for 5 + 1 and 1 + 5. For the "Calculator View" they should choose one fact pair and show each step from the view window. For the Number Line section, students use two colors to draw lines showing how they "count on" for each problem.

Calculator View Example for 5 +1 = 6 and 1 +5 = 6:

Pressing the (5) key: window shows 5; write 5 in the first box
Press + 1: window shows 1; write 1 in the second box
Press = window shows 6; write 6 in the third box
Repeat the process beginning with press (1) key.

After students complete the Think Sheet, have a discussion to share their information. *What did they discover about fact families? What relationships did they notice?*

Extension – Utilize the Commutative Property Think Sheet (SR4) - Challenge students to use two, three, or four digit addends to explore the commutative property employing larger numbers. See Re-teaching activity for general directions.

Lesson 2 – Is It Possible?
The Commutative Property and Subtraction, Multiplication, and Division

Pre-assessment - Give each student a 3x5 card with a basic fact problem (5 + 6; 9 – 3; 8 x 4; 12 ÷ 6). Ask students how they would solve their problem. They should be able to describe the method they used for solving the problem as well as the solution. For instance for 9 – 3: "I started at 9 and counted back 3, so the difference between 9 and 3 is 6." Use Teacher Observation Checklist (TR2) to record your observations.

Launch - *In our last lesson, we discovered that numbers can be added in any order, and sometimes that makes the computation easier. We called this idea the Commutative Property of Addition. Today, we will see if the commutative property works for the other operations.* Use whole class response such as "thumbs-up/thumbs-down" for the question: *Do you think the commutative property will work for subtraction?* Discuss why or why not. Then, look at an example such as 10 – 3 = 7. *What happens if we change the order of the numbers? What would it say?* (3 – 10 = 7) Continue by asking: *How might we read this (3 –10 = 7)? Three minus ten, how could we do that?* If students have an understanding that they can have an answer that is not a positive number, explain that the solution to the problem 3 – 10 will be a negative answer because 3 is less than 10 (3 – 10 = - 7). Repeat this process two more times using one division problem and one multiplication problem.

*Examples:

Using $10 \div 5 = 2$

$10 \div 5 = 2$

$5 \div 10 = 5/10$ or $1/2$

$10 \div 5 \neq 5 \div 10$

$2 \neq 1/2$

Using $5 \times 2 = 10$

$5 \times 2 = 10$

$2 \times 5 = 10$

$5 \times 2 = 2 \times 5$

$10 = 10$

Discuss students' observations based on the three examples. They should conclude that the commutative property probably works for multiplication, but not for subtraction and division.

Teacher Facilitation - Spend the rest of the lesson looking at multiplication models using pictorial and symbolic models. Begin by showing pictures such as those found in Multiplication Picture Stories (TR3). Compare each set of pictures equating them with their symbolic representations.

*Example: One picture representing 4 groups with 3 in each group or 4 x 3, and the compatible set representing 3 groups with 4 in each group or 3 x 4.

Determine whether the products are identical. Proceed through the three sets of pictures in the same manner. Then, discuss whether the commutative property worked for all examples. *Did the order matter?* Some confusion may result because the numbers are attached to specific elements within the pictures. However, the products are the same. It

is important to note that the question being answered by the product is "How many in all?"

Student Application – Students will continue their exploration of the commutative property of multiplication using an array model. For this activity, they may work individually or in pairs. Hand out Commutative Tile Company packet (cover - SR5; Floor Plans sheet - SR6; and Customer Recommendation Sheet - SR7). Present the following prompt and directions for completing SR6 and SR7:

> *You work for the Commutative Tile Company. You have been asked to provide an estimate of the cost for tiling several different floor plans. You have been given a sheet with the floor plan outlines, but the measurements are missing!*

Explain that the grids represent tile floors. The students' job will be to use color tiles to determine the amount of tiles needed to cover each floor. They must label the dimensions of the rooms and determine the area for each floor plan. Later, this information will be summarized on the Estimation Table. If needed, review the concept of area. Allow about time for partners to brainstorm how they might determine the area and complete the floor plans.

Reassemble the class and allow time for groups to share their floor plan results and describe their method of determining dimensions and area. Then, hold a class discussion. As the discussion progresses, record on a class chart the computations students used to determine the area.

Sample chart:

3 X 4 = 12	4 X 3 = 12
3 X 1 = 3	1 X 3 = 3
2 X 2 = 4	2 X 2 = 4

Then, continue with the above scenario giving the following instructions:

> *The Tile Company's customer is having difficulty deciding on a floor plan. Use the Estimation Table to compare the plans with equal areas. Decide which floor plan you would recommend to the customer and why you would recommend it. Be sure to explain your thinking on the bottom of the Customer Recommendation Sheet.*

Embedded Assessment – As students respond to the Tile Company questions note their responses. Do their explanations show their understanding of computing area? What does it tell you about their multiplication concept development?

Ask students to think back to the earlier discussion about the possibility of using the commutative property to solve multiplication problems. Looking at the chart just created in the previous activity, have them respond to the following questions:

- *Did the commutative property help them to solve the computation problems? How?*
- *What do they notice about the pairs of number sentences in the chart?*
- *Looking at the number sentences with the same area, are the problems the same? Why or why not?*
- *Does order matter when you multiply two numbers? Will this be true for any set of two numbers that are multiplied together?*

Collect the completed Commutative Tile Company packets. Evaluate student responses on the Customer Recommendation Sheet. Use Rubric for Commutative Tile Activity (TR4) to evaluate children's work.

Re-teaching - Give students a large card with a multiplication sentence written at the top. Have them outline the array representing the number sentence on a piece of Multiplication Array Paper (SR8). Cut it out and paste it on the side of the card that contains the number sentence. Tell them to turn the card over, reverse the numbers from the multiplication sentence, and write the new sentence on the blank side of the card. Now draw this array on the graph paper, cut it out, and glue it under the new number sentence. Discuss the students' observations about the two number sentences and their respective pictures.

Extension - Allow able students to investigate the commutative property using double-digit multiplication. Give them the following problem:

> The Bunyon family went shopping for computer cartridges. Brian Bunyon saw a sign advertising Blooble Cartridges. It said, "5 packages for $25.00". Brian's brother, Bob, saw another sign reading "Single cartridges $1.00 apiece (Must get 25 cartridges!)".

Discuss: *How are the deals alike? How are they different? Cost-wise, does it matter which deal they choose?*

Lesson 3: **Magic Association Operations**
What Is the Associative Property and How Is It Used?

Pre-assessment: Place the following problems on the board:

25 + 32 + 48 = ___	64 X 2 = ___
59 + 10 + 11 = ___	25 X 4 = ___

Allow time for students to copy and complete these problems. Tell them they can use any addition strategy that works for them. However, they will need to be able to explain the steps they employed to solve each problem.

Have several students come up to the board to explain their strategies. Note each child's response. As they describe their strategies, think about what number operation concepts they understand or are missing. Note these on Teacher Observation Sheet (TR 5).

Launch: Read the story <u>One Hundred Hungry Ants</u> by Elinor J. Pinczes. As you read, stop at the descriptions of how many ants are in each line. Elicit from the students an addition and a multiplication sentence for each set of lines.
*Examples:

2 lines of 50 write 50 + 50 = 100 and 50 X 2 = 100
4 lines of 25 write 25 + 25 +25 + 25 = 100 and 25 X 4 = 100
5 lines of 20 write 20 + 20 + 20 + 20 + 20 = 100 and 20 X 5 = 100

Complete the story, explain that solving these expressions can be accomplished using mental math. Ask: *Are these problems all easy to add/multiply?* Tell them that these are friendly numbers, but that is not always the case.

In this lesson, they will be introduced to a new mathematical property called the Associative Property. As they proceed through the lesson, they will discover how this property can help them solve addition and multiplication problems even if the numbers are not friendly.

Teacher Facilitation: Return to the Ants story scenario. Pretend the students are the ants. Call on 18 "ants" to begin their journey. The others are in the anthill. They play the roll of "look-out ants" watching for the groups return. Have students act out the following tale:

> *Ants 1, 2, and 3 are marching along looking for food. They meet another group of seven ants on their own hunt. They form one long line of <u>10</u>* (point to the group of "look-out ants" to shout out the total number in the line) *ants and continue along their way. Next, they find eight more ants busy as bees. These ants join the formation and head back to the anthill. Now there are* (all students reply) *<u>18</u> ants traveling in a line. The __* (number depends on how many students in the class) *"look-outs" see the long, marching line coming and welcome them home. Safe at home in the anthill are __ tired ants!*

Now, retell the story mathematically using numbers and symbols. Ask: *How can we begin? What can we put down first?* They should say to put a 3 to represent the 3 ants. They should continue by adding the addition symbol (+) and a 7. From here, the tendency would be to continue by adding another plus sign, an 8, an equal sign, and the number 18. Some classes will want to include the look-out ants. They would add a plus sign and then the number of look-outs. Before the look-out ants are added, the expression should look like this:

3 + 7 + 8 = 18

Look at this number sentence and think about the story. Remember that the group of three ants joined the group of seven ants. Then, there was a line of ten ants. On the

equation (3 + 7 + 8), how can we show that the three ants and the seven ants came together? They should suggest that parenthesis (younger students might refer to these as "huggy" symbols) could be put around the 3 + 7 part making the expression look like: (3 + 7) + 8 = 18. Complete the analysis of this equation:

$$(3 + 7) + 8 = 18$$
$$10 + 8 = 18$$
$$18 = 18$$

Continue by telling students to pretend the story changed so that another day, the three ants marched alone for a longer time. Meanwhile, the group of seven ants joined the group of eight ants to form one line. When the three ants arrived, there were eighteen ants traveling in a line. Elicit the new story and record as an equation. It will look like:

$$3 + 7 + 8 = 18$$
$$3 + (7 + 8) = 18$$
$$3 + 15 = 18$$
$$18 = 18$$

Have students compare the two representations. Ask them what they notice about the two number sentences and the solution. Allow them to discover that the way they combined the terms did not matter. The total remained the same. Explain that the mathematical name for the idea that grouping does not matter when adding numbers is the Associative Property.

Recall from previous lessons that the Commutative Property said that order did not matter when used for addition and multiplication. Explore a short multiplication example like these ant formations:

1. Begin with three lines of four ants who meet up with another group of three lines of four ants. How many ants are there if both groups of ants join together?

2. Three groups of ants are marching along. Each group consists of four lines - two ants per line. The three groups join together. Now, how many ants are there altogether?

If needed for clarification, draw pictures of the two stories. Then, elicit mathematical expressions for both of the stories.
Compare the expressions using multiplication:

3 x 4 x 2	verses	3 x 4 x 2
(3 x 4) x 2		3 x (4 x 2)
12 x 2		3 x 8
24		24

Ask: *What did you notice about the two number sentences and the solution?* Allow them to discover that the way they grouped the terms did not matter. The total remained the same. Ask them if this was similar to what they found out in the addition problem, reinforcing the idea that the Associative Property (the concept that grouping does not matter when finding a sum or product) works for multiplication as well as addition.

Student Application:
Give students a large blank card. Direct them to write their own short, addition or multiplication stories on one side of the card. On the other side, have them write the story as number sentences. Give students a maximum of 20 minutes to complete this task. Collect the cards without much discussion, and save for later use during the assessment activity.

Embedded Assessment:
Hand out the problems (completed in the previous session) to students other than the problem's author. Also, hand out another blank card. Challenge them to solve each other's story problems. On one side of the blank card, the student will write her/his solution strategy. On the other side, they will compare their solutions with the writer's solution. They will be assessed on their observations. Ask them how they might compare solutions. Create a class rubric of acceptable responses. These might include: An explanation of their solution strategy and an explanation of the writer's strategy; the reasoning behind their strategy; and a sentence explaining that the way groups are formed or combined does not matter when doing addition or multiplication. Allow 20 to 30 minutes for students to complete the assessment.

Re-teaching:
Give students other story problems. Let them use manipulatives such as snap cubes, color tiles, or Cuisenaire Rods to "act out" the stories. They might also draw pictures to represent the number sentences from the story. These concrete representations should be translated into written numeric equations. Some students will need several such experiences before internalizing the Associative Property.

Extension
Have students sort the problems they created into those that are fairly easy to solve and those that are more difficult. Have them discuss what attributes exist for both kinds of problems. Have students take a harder problem and use materials and/or diagrams to make it easier to understand. Place these into a center.

Summative Assessment:
Students will complete the Summative Assessment (SR9). Part A of the assessment is multiple choice. Students will read each question, look carefully at the four answer choices, and decide which answer is most appropriate. The questions in Part B are Brief

Constructed Response (BCR) questions. Some require brief written explanations based on reasoning. Others are two-part response questions. The first part asks students whether they agree or disagree. The response is in the form of a *yes* or *no* answer. In the second part, the student will use pictures, numbers, and words to explain their thinking. Use the Summative Assessment Rubric (TR5) to score each response separately.

Alternate Summative Assessment:
Students will complete an Assessment Project. If technology is available, they will develop a slide show using "*Kid Pix"* or a "*Power Point"* presentation explaining what they know and understand about the Commutative and Associative Properties.

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Teacher Check List
Counting and Number Relationship Strategies

Teacher Note:
Place a check in the box corresponding to the strategy used by each named student.
Assessment questions to think about:
- What counting strategy did the student use?
- Were they comfortable with counting on?
- What strategy was used to figure out how many numbers were counted?

Counting/Number Relationship Strategy	Counting On Counting back	Makes Ten	Doubles/ Doubles plus one	Number plus One Two Zero	Compares Amounts	Note a pattern/ see a pattern effect	Identifies Sequence	Other
Names								

Counting/Number Relationship Strategy	Counting On Counting back	Makes Ten	Doubles/ Doubles plus one	Number plus One Two	Compares Amounts	Note a pattern/ see a pattern	Identifies Sequence	Other
<u>Name</u>								

Teacher Observational Checklist

Counting Strategy/ Number Relationship Perspective	Name	Name	Name	Name	Name	Name	Name
Counts on/Counts Back By ones							
Makes Tens							
Doubles/Doubles plus one							
Number plus one/ Number plus two							
Compares amounts							
See part-part- whole relationship							
Use relationships between operations							

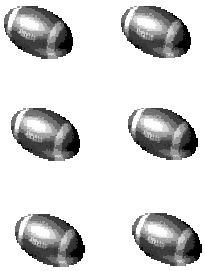
Anecdotal Information / Evaluation Scale

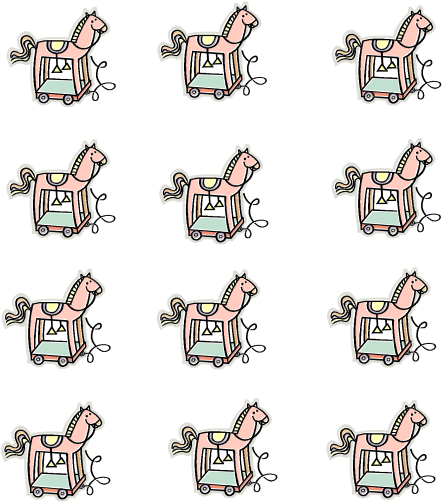
Place a check in the box under each student's name corresponding to the strategy used.

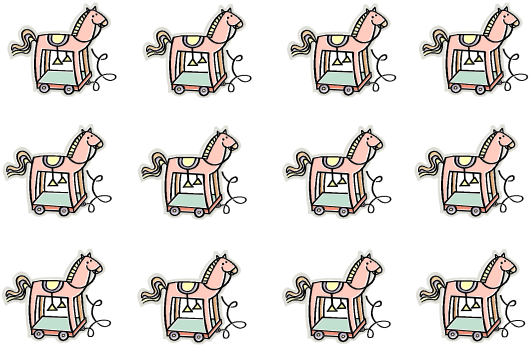
Assessment questions to think about:

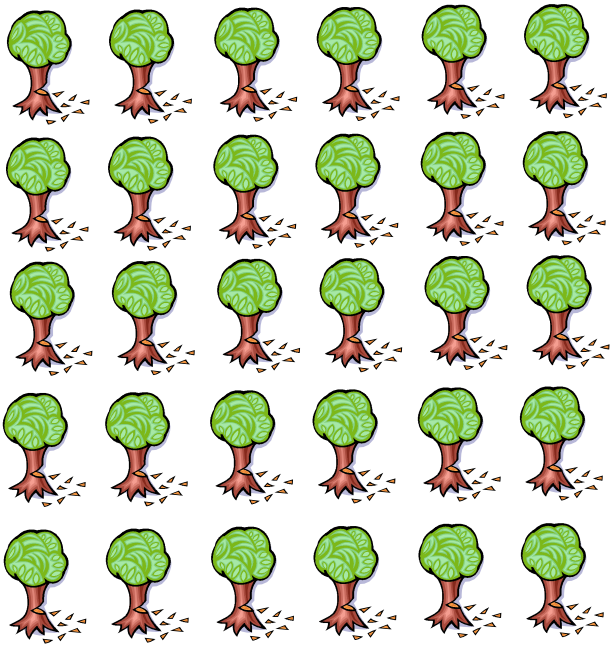
- What strategy did students use to solve the basic fact problem?
- What counting strategy did the student use?
- What strategy was used to figure out how many numbers were counted?

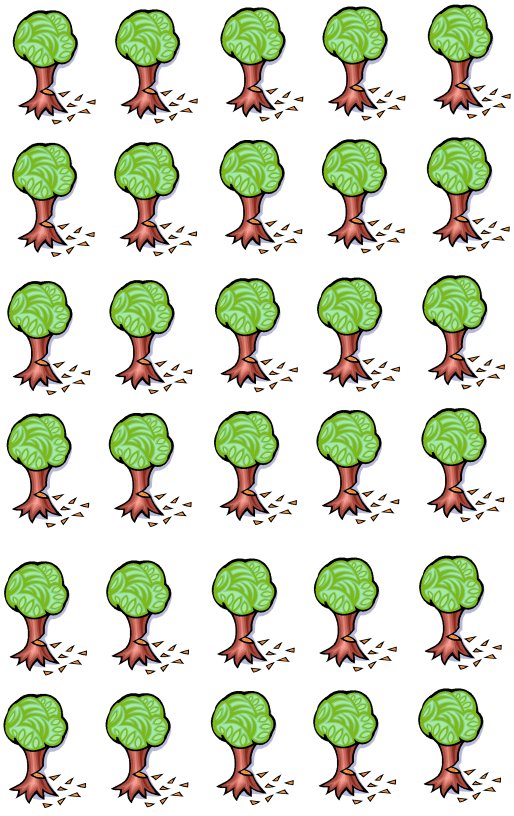
Multiplication Picture Stories











Rubric for Commutative Tile Company Packet

3 Points	o **Floor Plans have correct length and width listed for each plan** o **Accurate information on the Estimation Table** o **Explains, completely, advantages and disadvantages of equal area floor plans** o **Recommendation is logical**
2 Points	o **Floor Plans have correct length and width listed for each plan** o **Estimation Table is completed, but contains some inaccurate information** o **Limited Recommendation**
1 Point	o **Only Floor Plan is accurate.** o **All other information is incomplete or missing**
0 Points	o **Floor Plan contains incorrect lengths and widths** o **Packet is incomplete** o **All other information is incomplete or missing**

Teacher Observation Checklist

Addition/Multiplication Strategies	Name	Name	Name	Name	Name	Name	Name

Anecdotal Information / Evaluation Scale

In the left hand column, note strategies used. Place a check in the box corresponding to the strategy used by each named student.

Assessment questions to think about:

- What addition or multiplication strategy did students use to solve the problems? Adding on? Grouping? Compatible numbers?
- What number operation concepts do the children understand?
- What method did the students employ? Standard algorithm? Efficient invented algorithm?

Summative Assessment Rubric

Part A

Item	Outcome	Points	Criteria/Answer
1	Describe and make a generalization about a pattern	1 – Correct answer 0 – Incorrect answer	B (6 + 4 + 2) - This is an extending pattern so each shape represents a member of the total group.
2	Analyze number relationships and compare equivalent expressions	1 – Correct answer 0 – Incorrect answer	C (15 + 26 = 26 + 15) – This is an example of the commutative property of addition. It also represents an understanding of the equality and its symbolic representation.
3	Understand and use properties of operations	1 – Correct answer 0 – Incorrect answer	A (35 x 2 = 2 x 35) – This is an example of the commutative property of multiplication.

Part B

Item	Outcome	Points	Criteria/Answer
1	Discover flexible ways to add numbers	2 – Answers yes and explains another addition strategy 1 – Answers yes and gives a partial explanation of an addition strategy 0 – Incorrect answer	Maria explains the traditional addition algorithm. Responses may vary. Examples 3 + 7 = 10 so 83 + 17 = 100. Add 32 more and the total equals 132. 80 + 10 + 30 = 120; 3 + 2 + 7 = 12; 120 + 12 = 132
2	Understand and use properties of operations; Represent and analyze mathematical situations using symbols, words, and pictures	3 – Correct number in the blank and represents the story mathematically showing evidence of the associative property of addition 2 – Correct number in the blank and represents the story 1 – Correct number in the blank 0 – Incorrect answer	10 Birds on the fence Picture shows a group of 5 blue birds then 1 more blue bird and finally a group of 4 blue birds on or flying to a fence. (5 + 1) + 4 = 10

Part B (con't.)

Item	Outcome	Points	Criteria/Answer
3	Describe and make generalizations about numeric patterns; Understand and use properties of operations; Represent and analyze mathematical situations using symbols, words, and pictures	2 – Answers yes and provides a thorough explanation showing understanding of the commutative property of multiplication 1 – Answers yes and gives a partial explanation with some logical reasoning 0 – Incorrect answer	Yes – Justifications may vary. Sample responses • The first band can be represented by 3 rows with 4 in each row (3 x 4). The second band has 4 rows with 3 in each row (4 x 3). Therefore, they have the same total number (12) of band members. • 4 x 3 = 3 x 4 because 12 = 12

Roll an Order

Instructions: Player 1 generates a number and places it in the first box under "Player 1". Player 2 repeats the process, putting their number in the first box under Player 2. Each player completes her/his equation by placing their partner's number in the empty box next to the operation sign, solves their number sentence, and compares their sum with their partner's sum.

Player #1	**Player #2**
☐ + ☐ = ☐	☐ + ☐ = ☐
☐ + ☐ = ☐	☐ + ☐ = ☐
☐ + ☐ = ☐	☐ + ☐ = ☐
☐ + ☐ = ☐	☐ + ☐ = ☐

What patterns do you notice as you look at the number sentences?

_____

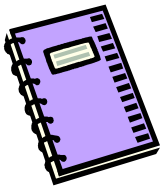
_____

_____

_____

Student Resource 2

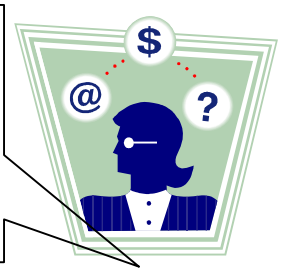
My Math Journal



Thinking about: _____

_____

_____



_____

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Scoring Criteria

☐ _____

☐ _____

☐ _____

☐ _____

Student Resource 3

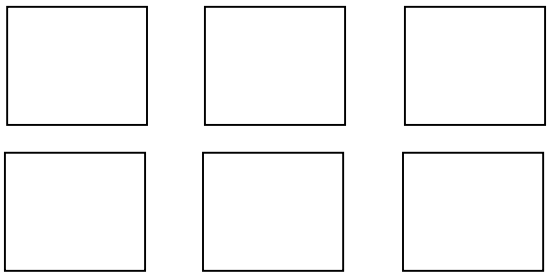
Think Sheet for Addition
Using the Commutative Property

Write the Number Sentences

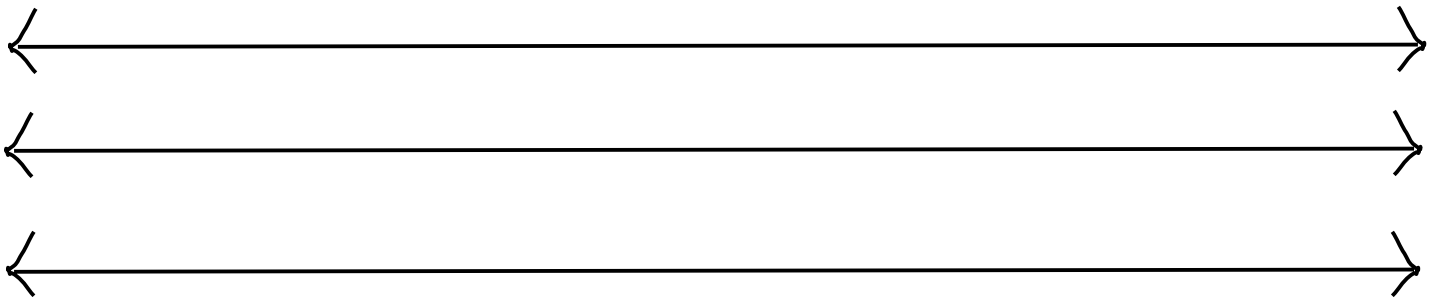
Addition Table

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4										
5										
6										
7										
8										
9										

Calculator View



Number Lines



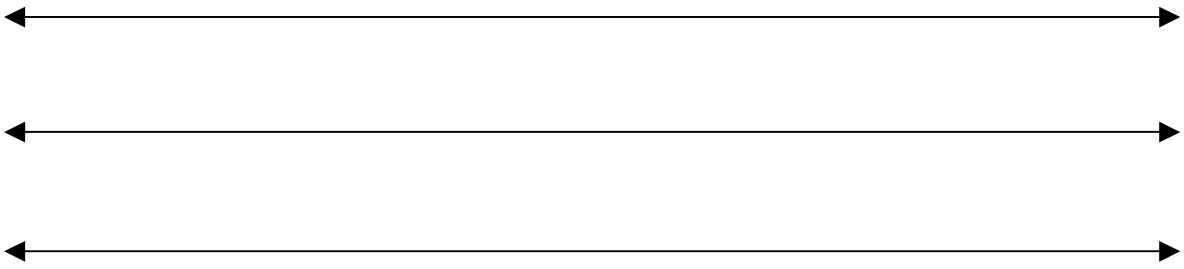
Commutative Property
Think Sheet

Write the Equation

Calculator View

	+		=			
	+		=			

Number Line

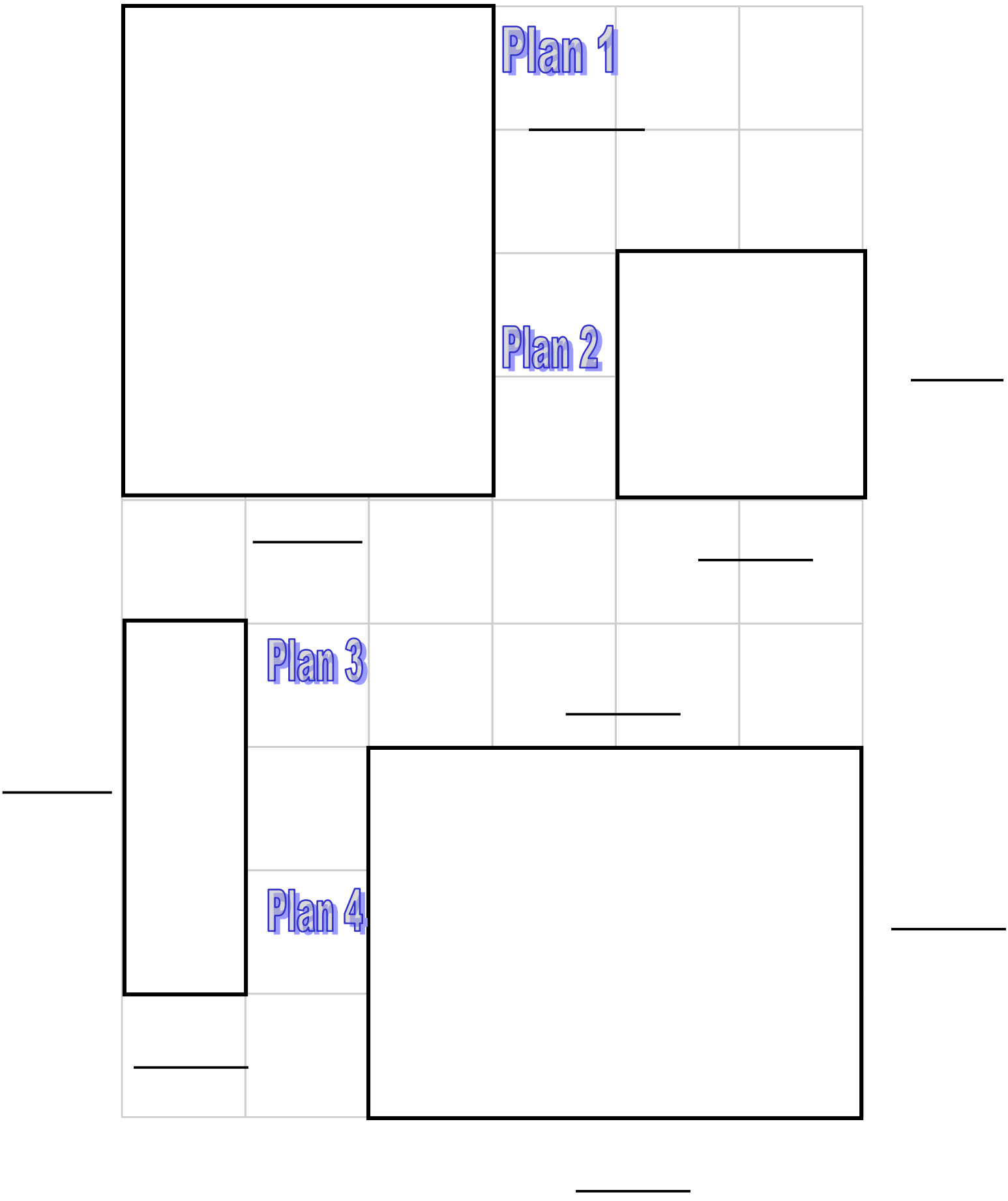


Commutative Tile Company Packet

NAME:

_____

Floor Plans



Commutative Tile Company
Customer Recommendation Sheet

Estimation Table

Plan #	Dimensions	Expressed as Multiplication	Area	Cost @ $5.00 a square unit

Comparing Plans of Equal Area

Plan ____

Length	Width	Area

Advantages/Disadvantages:

Plan ____

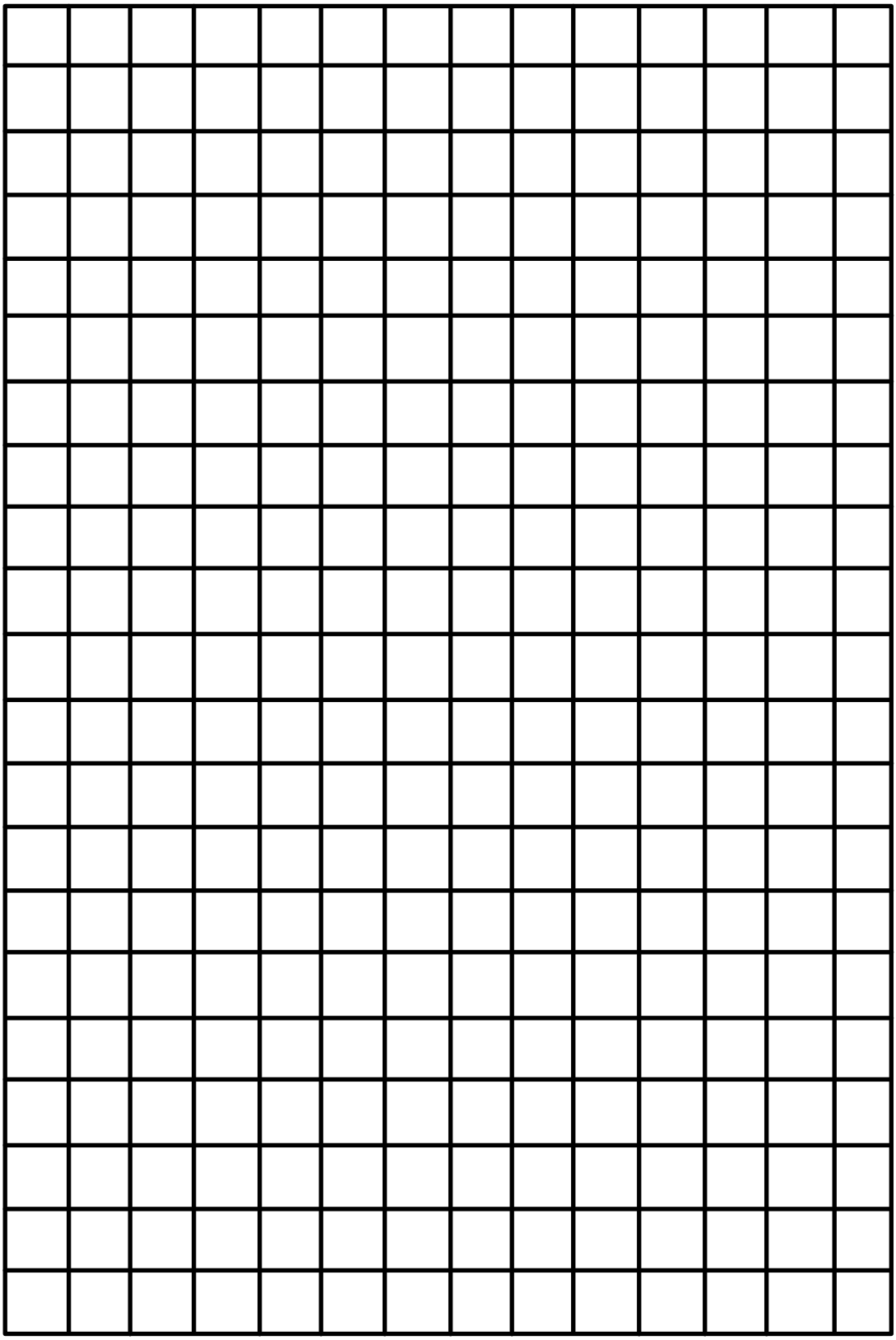
Length	Width	Area

Advantages/Disadvantages:

Recommendation:

_____
_____
_____
_____
_____
_____
_____
_____

Multiplication Array Paper



Name _____ *Student Resource 9*

Date _____

Summative Assessment

Part A

Circle the letter corresponding to your answer choice.

Look at the pattern:

1. Which expression best represents this pattern?
   a. 2 x 6
   b. 6 + 4 + 2
   c. 6,4,2
   d. 3 x 4 = 12

2. Find the number sentence that is written correctly.

   a. 15 + 26 = 27 + 15
   b. 89 + 4 = 894
   c. 15 + 26 = 26 + 15
   d. 15 + 26 = 15 x 26

3. Which problem is an example of the commutative property of multiplication?

   a. 35 x 2 = 2 x 35
   b. 35 x 2 = 35 x 2
   c. 48 x 3 = (40 + 8) x 3
   d. (12 + 3) = (10 + 2) + 3

Part B

1. Maria solved the problem 83 + 32 + 17. She got an answer of 132. When asked how she got that answer, she explained: First I looked at the ones column and added 3 + 2 and got 5. Then I counted on seven more and got 12. I put the 2 down and carried the 1. Next, I added the 1 and the 8 and got 9. I counted on 3 more to get 12. Then I added one more, so my answer is 132.

Do you agree with her answer? ____ Yes ____ No

Name _____ ***Student Resource 9***

Date _____

Maria's friend Sam solved the problem using a different strategy. He said that he liked to make tens, and he knew that 3 + 7 = 10. How might Sam use this knowledge to solve the problem?

2. Complete the story by filling in the blank.

5 blue birds flew towards a fence. Soon 1 more came to join them. 4 more blue birds arrived. A boy came. He looked at the fence and exclaimed, "There are _____ blue birds sitting on a fence!"

Draw a picture and write a number sentence to represent the story.

3. Look at the picture below. It represents two marching bands.

xxxx xxx
xxxx xxx
xxxx xxx
xxx

Do the two bands have the same number of members? ___ Yes ___ No

Justify your answer using pictures, words, and numbers.